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On the theory of acoustic waves in polydispersed gas–vapor-droplet suspensions

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Abstract

A linear theory on the propagation of plane waves in the polydispersed gas-droplets suspensions is presented. The case is considered when the gas carrier phase is a homogeneous mixture of two components. The first one is a vapor, and the second is a neutral gas. Both non-steady and non-equilibrium effects of the phase interaction (mass, momentum, energy interface exchange) are taken into account. A general dispersion equation is obtained. This equation describes the speed of propagation and attenuation of disturbances in the polydispersed gas–vapor-droplets suspensions with arbitrary droplet distribution functions by size. It holds true for a wide range of frequencies complying with the requirement for the medium acoustic homogeneity. An effect of phase transformations (evaporation and condensation) influenced by diffusion of vapor through the neutral gas is studied. Some calculations are done for polydispersed fogs consisting of air and water droplets. High and low frequency asymptotics of the complex wave number that give velocities of the waves and their attenuation are obtained and analyzed. A possibility is shown to describe adequately and simply the attenuation of high and low frequency disturbances in terms of the 'monodisperse' models with the use of some averaged droplet radii. A paradoxical effect is found consisting in the non-monotonous dependence of sound attenuation on mass content of droplets, which are the main cause for wave absorbtion. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Attenuation; Condensation; Diffusion; Dispersion; Distribution function; Evaporation; Fraction; Gas–vapor-droplet suspensions; Phase sound speed; Plane waves; Polydispersed mixture; Mass exchange

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1. Introduction

Most of the investigations on the acoustics of gas–vapor-droplets media are devoted to monodisperse systems (Cole and Dobbins 1970; Davidson 1975; Gubaidullin and Ivandaev, 1987, 1990, 1991; Gumerov et al., 1988; Marble, 1970; Marble and Candel, 1975; Nigmatulin et al., 1991; Shagapov, 1987). See also the book by Nigmatulin (1990).

One of the earliest publications on the acoustic waves in polydispersed suspensions (with no phase transitions) was that by Temkin and Dobbins (1966). Along with a number of other papers, this work presented a case for small contents in the dispersed phase, when the contribution of each particles' fraction, characterized by the radius of the droplets, a, to the dissipation and dispersion of the disturbances is proportional to their mass fraction. Later on, when studying the propagation of weak disturbances Ishii and Matsuhisa (1983) took into account finite droplet mass contents in polydispersed gas particle suspensions. Yet the latter paper, like the previous one, did not consider the influence of non-stationary effects of interface momentum and heat exchange. Gumerov and Ivandaev (1988) studied the propagation of linear monochromatic waves in the specific cases of a single-component polydispersed gas or vapor-droplet mixture taking into account non-steady processes of phase interactions for droplet random mass contents. In doing so, they did not analyze the case with vapor-droplet mixture. A detailed consideration was only given to the most common case of no interfacial mass transfer. The main objective of the present paper is to study sound propagation in real two-component vapor–gas-droplet systems with regard to the effect of phase transformation (evaporation and condensation) influenced by diffusion of vapor through the neutral gas. A detailed analysis made in the present paper shows that the essential influence of diffusive mass exchange results in the anomalous effect of non-monotonic dependence of wave attenuation on mass concentration of droplets, which are the main cause for disturbance dissipation.

2. Formulation of the problem

Let us consider a plane one-dimensional flow of a polydispersed vapor–gas-droplet mixture in the acoustic field. The dispersed or fraction content is characterized by the distribution function, $N(a)$, being a function of the radius of the droplets, a, with the minimum $a_{\min}$ and maximum $a_{\max}$ radii of the droplets:

$$\delta n = N(a)\,\mathrm{d}a,$$

$$N(a) = 0 \quad \text{for: } a < a_{\min} \quad \text{and} \quad a > a_{\max}\,. \tag{2.1}$$

Here δn is the number of droplets with the radius in the range, a, and $a + \mathrm{d}a$ per unit volume of the mixture. Due to phase transition (evaporation or condensation) the radius of the droplets for the fraction may change with the velocity:

$$\dot{a} = \dot{a}(a,t). \tag{2.2}$$

The total number of the droplets, n, per unit volume, and volume content of the dispersed phase, α_2, and carrier (gas) phase, α_1, are determined by the integrals:

$$n = \int_{a_{\min}}^{a_{\max}} N(a)\,\mathrm{d}a, \quad \alpha_2(t) = \frac{4}{3}\pi\int_{a_{\min}}^{a_{\max}} a^3 N(a)\,\mathrm{d}a, \quad \alpha_1 + \alpha_2 = 1. \tag{2.3}$$

Averaged parameters for the polydisperse phase, particularly the velocity, v_2, interface mass exchange rate, j, interfacial force, f, heat transfer, q_i, between the surface of the droplet and i-th phase (i = 1, 2) per droplet in the one-dimensional continua approximation are given in each point (x, t) and depend on the droplet radius, a, of the fraction:

$$v_2 = v_2(x,t,a), \quad j = j(x,t,a), \quad f = f(x,t,a), \quad q_i = q_i(x,t,a), \ldots. \tag{2.4}$$

The basic parameters of the mixture are the following:

$$\rho_1 = \rho^\circ{}_1\alpha_1, \quad \rho_2 = \rho^\circ{}_2\alpha_2 = \int_{a_{\min}}^{a_{\max}} g(a)N(a)\,\mathrm{d}a, \quad g(a) = \frac{4}{3}\pi a^3\rho^\circ{}_2$$

$$m = \frac{\rho_2}{\rho_1}, \quad k_j = \frac{\rho_j}{\rho_1} \quad (j = V,G), \quad k_V + k_G = 1. \tag{2.5}$$

Here $\rho^\circ{}_i$, ρ_i are the true and reduced densities of the gas phase (i = 1) and the particles (i = 2), respectively; g is the mass of a single droplet; m is the mass content of the droplets; k_j is the initial mass concentration of the vapor ($j = V$) and gas ($j = G$) components of the gas phase. Note that the case of single-component mixtures of gas ($k_V = 0$) or vapor ($k_V = 1$) with droplets was discussed previously by Gumerov and Ivandaev (1988).

Here we restrict ourselves to the study of the initially uniform equilibrium state of the mixture along the x-coordinate, when the parameters with subscripts 0 ($\psi_0 \equiv \rho_{10}$, ρ_{20}, $v_{10} = v_{20}$, p_{10}, and so on) are constant by x and by time t, and consider small plane disturbances marked with prime, ψ':

$$\psi = \psi_0 + \psi', \quad \psi' \ll \psi_0$$

$$(\psi \equiv \rho_1, \rho_2, v_1, v_2, p_1, i_V, i_G, u_2, \ldots). \tag{2.6}$$

Linearized equations for these one-dimensional disturbances in a monodispersed fog follow from the general equations of motion for two-phase mixtures (Nigmatulin, 1990). They take the form (Gubaidullin and Ivandaev, 1987):

$$\frac{\partial\rho_1'}{\partial t} + \rho_{10}\frac{\partial v_1'}{\partial x} = -J, \quad \frac{\partial\rho_V'}{\partial t} + \rho_{V_0}\frac{\partial v_1'}{\partial x} = -J, \quad \frac{\partial\rho_2'}{\partial t} + \rho_{20}\frac{\partial v_2'}{\partial x} = J,$$

$$\rho_{10}\frac{\partial v_1'}{\partial t} + \frac{\partial p_1'}{\partial x} = -F, \quad \rho_{20}\frac{\partial v_2'}{\partial t} = F,$$

$$\rho_{V_0}\frac{\partial i'_V}{\partial t}+\rho_{G_0}\frac{\partial i'_G}{\partial t}=\alpha_{10}\frac{\partial p'_1}{\partial t}-nq_1,\quad \rho_{20}\frac{\partial u'_2}{\partial t}=-nq_2,$$

$$q_1+q_2=-jl_0\quad (J=nj,\quad F=nf). \tag{2.7}$$

Here p_1 is the pressure of the gas phase (gas and vapor), i_j is the specific enthalpy of the vapor $(j=V)$, gas $(j=G)$, u_2 is the specific internal energy of the liquid droplets, l is the specific evaporation heat.

The coordinate system is connected with the undisturbed equilibrium mixture:

$$v_{10}=v_{20}=0,\quad T_{10}=T_{20},\quad p_{V_0}=p_S(T_0), \tag{2.8}$$

where T_i is the temperature of the i-th phase, p_{V_0} is the vapor partial pressure in the gas phase in the initial state; $p_S(T_0)$ is the vapor saturation pressure corresponding to the initial temperature, T_0.

A set of linear differential equations of motion in the polydispersed vapor–gas-droplet mixture should be obtained through integrating the fractional linearized mass, momentum and energy equations (Nigmatulin et al., 1991; Gubaidullin and Ivandaev, 1987) by the droplet radius, a, going from $a_{\min}$ to $a_{\max}$.

For example, let us obtained the conservation equation of mass for the whole dispersed (droplet) phase supposing that the motion of droplet fraction with the droplet radius from a to $a+\delta a$ is described as a motion of monodisperse media with characteristic radius a. Then $\delta\rho_2=g(a)N(a)\delta a$ is the reduced density of this droplet fraction, while $\delta\rho_{20}=g(a)N_0(a)\delta a$ is the reduced density of this fraction in the initial state, and $\delta\rho'_2$ is the disturbance of the reduced density: $\delta\rho_2=\delta\rho_{20}+\delta\rho'_2$. In this case:

$$\int_{a_{\min}}^{a_{\max}}\delta\rho_2(x,t,a)\,\mathrm{d}a=\rho_2,\quad \int_{a_{\min}}^{a_{\max}}\delta\rho_{20}(x,t,a)\,\mathrm{d}a=\rho_{20},\quad \int_{a_{\min}}^{a_{\max}}\delta\rho'_2(x,t,a)\,\mathrm{d}a=\rho'_2. \tag{2.9}$$

The linearized conservation equation of mass for the fraction having non-disturbed reduced density $\delta\rho_{20}$ is identical to the third equation of (2.7):

$$\frac{\partial}{\partial t}[\delta\rho'_2(x,t,a)]+\delta\rho_{20}\frac{\partial}{\partial x}[v_2(x,t,a)]=jN_0\delta a. \tag{2.10}$$

On integrating this equation by the droplet radius a going from $a_{\min}$ to $a_{\max}$ one obtains

$$\frac{\partial\rho'_2}{\partial t}+\int_{a_{\min}}^{a_{\max}}\left[\frac{\partial v'_2(x,t,a)}{\partial x}N_0(a)g_0(a)\right]\mathrm{d}a=\int_{a_{\min}}^{a_{\max}}j(x,t,a)N_0(a)\mathrm{d}a. \tag{2.11}$$

A set of linear differential equations of motion in the polydispersed vapor–gas-droplet mixture is analogous.

Finally, the equations of mass and momentum conservation of the carrier phase and dispersed droplets have the following form

$$\frac{\partial\rho'_i}{\partial t}+\rho_{i_0}\frac{\partial v'_1}{\partial x}=-J,\quad \frac{\partial\rho'_2}{\partial t}+\int_{a_{\min}}^{a_{\max}}\frac{\partial v'_2}{\partial x}N_0g_0\,\mathrm{d}a=J,$$

$$\left(J=\int_{a_{\min}}^{a_{\max}} N_0 j\,\mathrm{d}a,\quad i=1,V\right)$$

$$\rho_{10}\frac{\partial v_1'}{\partial t}+\frac{\partial p_1'}{\partial x}+\int_{a_{\min}}^{a_{\max}} fN_0\,\mathrm{d}a=0,\quad g_0\frac{\partial v_2'}{\partial t}=f,$$

$$\rho_1=\rho_V+\rho_G,\quad p_1=p_V+p_G, \tag{2.12}$$

where J is the total interface mass exchange rate per unit volume determined by the vapor diffusive flow towards the droplet surface and by the condensation or evaporation rate on the surface of an individual droplet; f is the interface force per droplet by the carrier phase. Here the disturbance of the parameters are primed.

Below is the constant specific heat approximation for enthalpy of gas and vapor and for the internal energy of the liquid:

$$i_V=c_{pV}T+i_{V_0},\quad i_G=c_{pG}T+i_{G_0},\quad u_2=c_2T+u_{20}, \tag{2.13}$$

where c_{pV} and c_{pG} are the specific heat capacity under constant pressure for vapor and gas, c_2 is the specific heat capacity of the incompressible dispersed phase. Then the equations of internal energy for the gas phase, droplet fractions and their interface will be put down as

$$\rho_{10}c_{p_1}\frac{\partial T_1'}{\partial t}-d_{10}\frac{\partial p_1'}{\partial t}=-\int_{a_{\min}}^{a_{\max}} N_0 q_1\,\mathrm{d}a,\quad g_0c_2\frac{\partial T_2'}{\partial t}=-q_2,$$

$$q_1+q_2=-jl_0,\quad (p_1'=p_V'+p_G',\ c_{p_1}=k_Vc_{pV}+k_Gc_{pG}). \tag{2.14}$$

As a whole the linearized equations of vapor and gas mixture state may be put down in the following form:

$$p_V'=\frac{C_V^2}{\gamma_V\alpha_{10}}\rho_V'+p_{V_0}\frac{T_1'}{T_0},\quad p_1'=\frac{C_{10}^2}{\gamma_{10}\alpha_{10}}(\rho_1'+\Delta\bar{R}(\rho_V'-k_V\rho_1'))+p_{10}\frac{T_1'}{T_0},$$

$$\Delta\bar{R}=(R_V-R_G)/R_{10},\quad (R_{10}=k_VR_V+k_GR_G). \tag{2.15}$$

Here R_k, C_k and γ_k are the gas constant, sound speed and adiabatic exponent in the k-th component (k = 1, G, V).

The force exerted upon a particle of a dispersion mixture may be predetermined in the form of a sum of the quasistationary force of viscous friction f_μ (Stokes force), Archimedes force F_A, force of associated mass f_m and Basset force f_B:

$$f=f_\mu+f_A+f_m+f_B,$$

$$f_\mu=6\pi a\mu_1(v_1-v_2),\quad f_A=\frac{4}{3}\pi a^3\rho^\circ{}_1\frac{\partial v_1}{\partial t},$$

$$f_m = \frac{2}{3}\pi a^3 \rho^\circ{}_1 \frac{\partial}{\partial t}(v_1 - v_2),$$

$$f_B = B\int_{-\infty}^{t} \frac{\partial}{\partial \tau}(v_1 - v_2)\frac{\partial \tau}{\sqrt{t-\tau}}, \quad \left(B = 6a^2\sqrt{\pi \rho^\circ{}_1 \mu_1}\right), \tag{2.16}$$

where μ_1 is the coefficient of the viscosity of the gas phase.

In the case of weak monochromatic waves in suspensions, v_1 and v_2 are represented by complex exponents of coordinates and time $v_i = A_i \exp i(kx - \omega t)$, where A_i are the complex amplitudes of the phase velocity v_i (i = 1, 2) fluctuations. This enables us to easily calculate the integral in the expression for the Basset force by reducing it to the well-known probability integral (Nigmatulin, 1990, p. 148).

The heat fluxes q_i (i = 1, 2) to the interface and the interface diffusive flux j are determined by the following formulas (Nigmatulin, 1990, p. 164):

$$q_i = 2\pi a \lambda_i Nu_i (T_i - T_\Sigma), \quad j = 4\pi a^2 D_1 Sh_1 \frac{k_V - k_{V\Sigma}}{2a}, \tag{2.17}$$

where λ_1 is the coefficient of the thermal conductivity of the gas phase, D_1 is the coefficient of mass inter-diffusion of the gas and the vapor, λ_2 is the thermal conductivity of the liquid in drops.

The correspondent relations $Nu_i(\omega)$ (i = 1, 2) and $Sh_1(\omega)$ have the form (Nigmatulin, 1990, p. 164; Gubaidullin, 1987):

$$Nu_1 = 2(1 + z_1) = \frac{2}{\eta_1(z_1)}, \quad Nu_2 = \frac{10}{\eta_2(z_2)},$$

$$Sh_1 = 2(1 + y) = \frac{2}{\eta_1(y)}, \quad \eta_1(y) = \frac{1}{1 + y}, \quad (|\ \eta_1\ |, |\ \eta_1\ | \le 1)$$

$$\eta_1(z_1) = \frac{1}{1 + z_1}, \quad \eta_2(z_2) = \frac{5[3z_2 - (3 + z_2^2)\tanh\ z_2]}{z_2^2(\tanh\ z_2 - z_2)},$$

$$y = \frac{1 - i}{\sqrt{2}}(\omega \tau_d)^{1/2}, \quad z_j = \frac{1 - i}{\sqrt{2}}(\omega \tau_{\lambda_j})^{1/2}, \quad (j = 1{,}2);$$

$$\tau_d = \frac{a^2}{D_1}, \quad \tau_{\lambda_1} = \frac{a^2}{\kappa_j} \quad \left(\kappa_j = \frac{\lambda_j}{\rho^\circ{}_j c_j}\right). \tag{2.18}$$

The intensity of non-equilibrium condensation on the interface is determined by the well-known Hertz–Knudsen–Langmuir formula (Nigmatulin, 1990, p. 69; Nigmatulin, 1978):

$$j = \frac{\beta}{\sqrt{2\pi R_V T_S}}(p_{V\Sigma} - p_{VS}), \quad (\beta \le 1,\ T_S(p_V)) \tag{2.19}$$

where β is the accommodation (condensation) coefficient.

We shall consider monochromatic disturbances (see below) with a frequency, ω. In general, the interface interaction intensity terms depend on vibration frequency, ω:

$$f = g_0 \frac{v_1' - v_2'}{\tau_v^*} - \frac{v_1'}{\tau_A^*}, \quad j = g_0 \frac{r}{p_{10}} \frac{p_V' - p_{V\Sigma}'}{\tau_{k_1}^*} = g_0 \frac{r}{p_{10}} \frac{p_{V\Sigma}' - p_{VS}'}{\tau_\beta},$$

$$q_1 = g_0 \frac{c_{p_1}}{m} \frac{T_1' - T_\Sigma'}{\tau_{T_1}^*}, \quad q_2 = g_0 c_2 \frac{T_2' - T_\Sigma'}{\tau_{T_2}^*}, \quad p_{VS}' = T_\Sigma' \frac{l_0 \rho^\circ{}_{V_0}}{T_0}$$

$$(r = \rho^\circ{}_{10}/\rho^\circ{}_{20}). \tag{2.20}$$

Here $p_{VS}(a, x, t)$ is the partial pressure of vapor saturation related to the temperature of the droplet surface $T_\Sigma(a, x, t)$ with the Clapeyron–Clausius equation; τ_ψ^* ($\psi = v, A, k_1, \beta, T_1, T_2$) are the complex time characterizing interface interaction intensity of an individual droplet with a gas phase in a high frequency acoustic field (Gubaidullin and Ivandaev, 1987; Gumerov et al., 1988; Nigmatulin, 1990, p. 345):

$$\tau_v^* = t_v \left[1 + \frac{1-i}{\sqrt{2}}(\omega\tau_{\mu_1})^{1/2} - \frac{1}{9} i\omega\tau_{\mu_1}\right]^{-1}, \quad \tau_A^* = -\frac{i}{r\omega}, \quad \tau_{k_1}^* = \frac{1}{3} \frac{R_V}{R_{10}} (1 - k_V) \frac{\tau_d}{1 + y},$$

$$\tau_\beta = \frac{1}{3} \frac{\sqrt{2\pi}}{\gamma_V} \frac{\gamma_1 C_V a}{\beta C_1^2}, \quad \tau_{T_1}^* = \frac{1}{3} \frac{\alpha_{10}}{\alpha_{10}} \frac{\tau_{\lambda_1}}{1 + z_1}, \quad \tau_{T_2}^* = \frac{1}{3} \tau_{\lambda_2} \frac{5[3z_2 - (3 + z_2^2)\tanh\ z_2]}{z_2^2(\tanh\ z_2 - z_2)},$$

$$\tau_v = \frac{2}{9} \frac{\rho^\circ{}_2 a^2}{\mu_1}, \quad \tau_{\mu_1} = \frac{\rho^\circ{}_1 a^2}{\mu_1}, \quad \tau_d = \frac{a^2}{D_1}, \quad \tau_{\lambda_1} = \frac{a^2}{\kappa_j} \quad \left(\kappa_j = \frac{\lambda_j}{\rho^\circ{}_j c_j}\right),$$

$$y = \frac{1-i}{\sqrt{2}}(\omega\tau_d)^{1/2}, \quad z_j = \frac{1-i}{\sqrt{2}}(\omega\tau_{\lambda_j})^{1/2}, \quad (j = 1{,}2); \tag{2.21}$$

Note that the expression for τ_v^* and τ_A^* takes into account the Stokes, Basset, Archimedes and associated mass forces.

The set of Eqs. (2.12)–(2.15), (2.20) and (2.21) is closed and can be used to investigate the propagation acoustic perturbations in polydispersed vapor–gas-droplet mixtures.

3. Dispersion equation

Let us study the fundamental solutions of the derived system of linear equations, which have the form of travelling monochromatic waves for perturbations

$$\psi' = A_\psi \;\exp\; i(K_* x - \omega t) = A_\psi \, \mathrm{e}^{-K_{**}x} \exp[i(Kx - \omega t)], \quad (i^2 = -1)$$

$$K_* = K + iK_{**}, \quad C^{(p)} = \frac{\omega}{K}, \quad C^{(g)} = \frac{\mathrm{d}\omega}{\mathrm{d}K}, \quad \sigma = 2\pi\frac{K_{**}}{K}, \tag{3.1}$$

where A_ψ is the complex amplitude of perturbation of the parameter ψ [see (2.6)], K_* is the complex wave number, K_{**} is the linear attenuation coefficient; $C^{(p)}$, $C^{(g)}$ and σ denote the phase velocity, group velocity and non-dimensional attenuation coefficient per wavelength.

Substituting solution (3.1) for Eqs. (2.12)–(2.17) we get the following relations:

$$-i\omega A_{\rho_k} + i\rho_{k_0} K_* A_{v_1} + \frac{r\rho_{20}}{p_{10}}\left\langle \frac{A_{p_V} - A_{p_{V\Sigma}}}{\tau^*_{k_1}} \right\rangle = 0 \quad (k = 1, V),$$

$$-i\omega A_{\rho_2} + i\rho_{20} K_* \langle A_{v_2} \rangle - \frac{r\rho_{20}}{p_{10}}\left\langle \frac{A_{p_{V\Sigma}} - A_{p_{VS}}}{\tau_\beta} \right\rangle = 0,$$

$$-i\omega\rho_{10} A_{v_1} + iK_* A_{p_1} + \rho_{20}\left\langle \frac{A_{v_1} - A_{v_2}}{\tau^*_v} \right\rangle = 0,$$

$$-i\omega\rho_{10} c_{\mathrm{p}_1} A_{T_1} + i\omega\alpha_{10} A_{p_1} + r\rho_{20} c_{\mathrm{p}_1}\left\langle \frac{A_{T_1} - A_{T_\Sigma}}{\tau^*_{\Sigma_1}} \right\rangle = 0,$$

$$-i\omega A_{v_2} - \frac{A_{v_1} - A_{v_2}}{\tau^*_v} = 0, \quad -i\omega A_{T_2} + \frac{A_{T_2} - A_{T_\Sigma}}{\tau^*_{T_2}} = 0,$$

$$rc_{\mathrm{p}_1}\frac{A_{T_1} - A_{T_\Sigma}}{\tau^*_{\Sigma_1}} + c_2 \frac{A_{T_2} - A_{T_\Sigma}}{\tau^*_{T_2}} + \frac{rl_0}{p_{10}}\,\frac{A_{p_{V\Sigma}} - A_{p_{VS}}}{\tau_\beta} = 0,$$

$$A_{p_V} - \frac{C^2_V}{\gamma_V \alpha_{10}} A_{\rho_V} - \frac{p_{V_0}}{T_0} A_{T_1} = 0, \quad A_{p_1} - \frac{C^2_{10}}{\gamma_{10}\alpha_{10}}[(1 - k_V \Delta\bar{R}) A_{\rho_1} + \Delta\bar{R} A_{\rho_V}] - \frac{p_{10}}{T_0} A_{T_1} = 0,$$

$$\left(\frac{A_{p_V} - A_{p_{V\Sigma}}}{\tau^*_{k_1}} = \frac{A_{p_{V\Sigma}} - A_{p_{VS}}}{\tau_\beta}, \quad A_{p_{VS}} = \frac{l_0 \rho^\circ{}_{V_0}}{T_0} A_{T_\Sigma}, \quad \tau^*_{\Sigma_1} = \frac{\alpha_{20}}{\alpha_{10}} \tau^*_{T_1} = \frac{1}{3}\,\frac{\tau_{\lambda_1}}{1 + z_1} \right) \tag{3.2}$$

where $\langle h \rangle$ denotes the linear operator of averaging by mass of the droplets:

$$\langle h \rangle = \frac{1}{\rho_{20}} \int_{a_{\min}}^{a_{\max}} N_0 g_0 h \, \mathrm{d}a. \tag{3.3}$$

Since the gas phase parameters are independent of radius, a, the following identity takes place:

$$\langle A_{\psi_1}\rangle \equiv A_{\psi_1}. \tag{3.4}$$

Let us write the unknown amplitudes, $A_{p_{V\Sigma}}$, $A_{p_{VS}}$, A_{T_Σ}, A_{v_2}, in terms of gas phase parameter amplitudes A_{ψ_1} ($\psi_1 = v_1$, T_1, p_V), which are independent of a and known characteristic times, τ^*_ψ. This allows us to factor out these unknown amplitudes from the sign of the averaging operator $\langle\ldots\rangle$:

$$A_{v_2} = \frac{A_{v_1}}{1 - i\omega\tau^*_v}, \quad A_{p_{V\Sigma}} = \frac{\tau_\beta A_{p_V} - \tau^*_{k_1} A_{p_{VS}}}{\tau^*_{k_1} + \tau^*_\beta},$$

$$A_{p_{VS}} = \frac{l_0\rho^\circ{}_{V_0}}{T_0} A_{T_\Sigma}, \quad A_{T_\Sigma} = Z\left[A_{T_1} + i\omega\tau^*_{\Sigma_1} e_2\left(\frac{l_0}{p_0 c_{\mathrm{p}_1}}\right)A_{p_V}\right],$$

$$Z = r\left[r - i\omega\tau^*_{\Sigma_1}\left(e_1 - \frac{\rho^\circ{}_{V_0}}{p_{10}} r l_0(\gamma_1 - 1)e_2\right)\right]^{-1}, \quad e_1 = \frac{c_2}{c_{\mathrm{p}_1}}\frac{1}{1 - i\omega\tau^*_{T_2}},$$
$$e_2 = \frac{1}{i\omega(\tau^*_{k_1} + \tau_\beta)}. \tag{3.5}$$

Substituting expressions (3.5) for (3.2) we shall obtain a uniform set of linear algebraic equations for the amplitudes, A_ψ ($\psi = \rho_1$, ρ_2, v_1, p_1, T_1, T_2, $\ldots$).

The dispersion equation connecting the wave number, K_*, and perturbation frequency, ω, can be derived from the condition of the existence of the non-zero solution (3.1) for the system of linear equations considered. The dispersion equation has the following form:

$$\left(\frac{C_1 K_*}{\omega}\right)^2 = V(\omega)D(\omega). \tag{3.6}$$

Here $V(\omega)$, $D(\omega)$ are the complex functions describing the effects of sound dispersion and dissipation in the suspension because of interface friction and interface heat and mass exchange correspondingly. If the particles are absent ($m = 0$, $V(\omega) = D(\omega) = 1$), there is neither dispersion nor dissipation in single-phase gas without droplets or particles. The functions $V(\omega)$, and $D(\omega)$ depend on the frequency, phase thermophysical parameters and spectral composition of the mixture by means of the functions given below

$$V(\omega) = 1 + m\frac{(\alpha_1 - r)\langle h_v\rangle - \alpha_1 r}{1 + mr\langle h_v\rangle}, \quad h_v = \frac{1 - \tau^*_v/\tau^*_A}{1 - i\omega\tau^*_v},$$

$$D(\omega) = 1 + m(\gamma_1 - 1)\frac{H_2 - \bar{R}_V k_V \gamma_1(\bar{R}_V \bar{c}_1 H_3 - 2\bar{l} r H_1) - M_1 \Lambda}{1 + m(H_2 - BH_3 - M_2\Lambda)},$$

$$\Lambda = rLH_1^2 + H_2 H_3,$$

$$H_j = \langle h_j \rangle, \quad j = 1{,}2{,}3: \quad h_1 = Ze_2, \quad h_2 = Z(e_1 - Le_2), \quad h_3 = Ze_2(r - i\omega\tau^*_{\Sigma_1} e_1),$$

$$B = (1 - \bar{R}_V k_V)\bar{R}_V, \quad L = r\gamma_1(\gamma_1 - 1)k_V \bar{l}^2, \quad M_1 = m\bar{R}_V \bar{c}_1(\gamma_1 - 1 + \bar{R}_V k_V), \quad M_2 = mB,$$

$$\bar{R}_V = \frac{R_V}{R_{10}}, \quad \bar{l} = \frac{l_0}{C^2_{10}}, \quad \bar{c}_1 = \frac{c_{p_1}}{\gamma_1 R_1} = \frac{1}{\gamma_1 - 1}, \quad \bar{c}_2 = \frac{2}{\gamma_1 R_1}. \tag{3.7}$$

This dispersion dependencies (3.6) and (3.7) is obtained for the case of small contents of droplets by volume ($\alpha_2 \ll 1$) and moderate pressures, when the ratio of the gas and liquid densities is small, $r = \rho^\circ{}_{10}/\rho^\circ{}_{20} \ll 1$. However, the droplets content by mass may be large enough. The account for the members with α_2 and r leads to the occurrence of multipliers in the form of $(1-\alpha_2)$ and $(1-r)$ in the dispersion equation.

The form of the dispersion relations (3.6) and (3.7) agrees with the corresponding dependencies by Gumerov and Ivandaev (1988) in the particular cases of the single-component vapor-droplet mixtures ($k_V = 1$) and gas particle mixtures with no phase transformations ($k_V = 0$, $\tau_\beta = \infty$). A particular case of the dispersion relation for the monodisperse vapor–gas-droplet mixture (Gubaidullin and Ivandaev, 1987) is obtained from (3.5) and (3.6), when substituting $N_0(a) = n_0\delta(a - a_0)$ for δ as the Dirac delta function, and then

$$\langle h_j \rangle = h_j(a_0,\omega) \quad (j = 1{,}2{,}3).$$

4. High and low frequency asymptotics

To get explicit expressions for the propagation velocities $C^{(p)}$, $C^{(g)}$ and the attenuation coefficients, K_{**} and σ, there is reason to analyze both high ($\omega\tau_v \gg 1$) and low ($\omega\tau_v \ll 1$) frequency asymptotics of the complex wave number K_*, following from the dispersion relationships (3.5) and (3.6), when the mass and volume concentration of the droplets are small: $m^3 \ll 1$, $\alpha_2 \ll 1$. Let us ignore the non-equilibrium state of the interface surface at the mass exchange ($\tau_\beta = 0$) and assume that the gas constants of vapor ($k = V$) and gas ($k = G$) are of the same order of values; the same is for the temperature diffusivity, kinematic viscosity and diffusion coefficients of the gas phase:

$$R_V \sim R_G, \quad \kappa_1 \equiv \frac{\lambda_1}{\rho^\circ{}_1 c_1} \sim \frac{\mu_1}{\rho^\circ{}_1} \sim D_1. \tag{4.1}$$

Further we shall use the following dimensionless parameters characterizing the aerosol thermophysical and acoustic properties

$$Pr_1 = \frac{\mu_1 c_{p_1}}{\lambda_1} \sim 1, \quad \bar{K}_* = K_* C_1 \tau^\circ{}_v, \quad \Omega = \omega\tau^\circ{}_v,$$

$$\bar{C}^{(\mathrm{P})} = \frac{C^{(\mathrm{P})}}{C_1} \quad \bar{a}_{i,j} = \frac{a_{i,j}}{a^\circ} \quad \left(\tau^\circ{}_v = \frac{2}{9}\frac{\rho^\circ{}_2 (a^\circ)^2}{\mu_1}\right) \tag{4.2}$$

where a° is some averaged scale radius of the droplets, which is determined below.

Within the equilibrium mass exchange approximation at any frequencies, ω, the temperature of the droplet surface, T_Σ, always remains equal to that of the saturation, $T_S(p_V)$, corresponding to the vapor partial pressure, p_V. For this case the high frequency ($\omega\tau_v \gg 1$) asymptotic of the dimensionless wave number $\bar{K}_*$ has the following form

$$\frac{\bar{K}_*}{\Omega} = 1 + \frac{m}{2}\{(1+i)\sqrt{r}K^{(1)}_\infty(\bar{a}_{3,2}\Omega)^{-1/2} + iK^{(2)}_\infty(\bar{a}_{3,1}\Omega)^{-1}\} + O(m^2,\Omega^{-2}).$$

$$(\Omega \gg 1) \tag{4.3}$$

The coefficients $K^{(1)}_\infty$ and $K^{(2)}_\infty$ of the first (main) and second terms of the asymptotic are determined by the formulae:

$$K^{(1)}_\infty = \frac{3}{2} + (\gamma_1 - 1)Pr_1^{-1/2}\frac{k_G\bar{c}_1 + k_V E_1}{E_2}, \quad K^{(2)}_\infty = 1 + \frac{2}{3}(\gamma_1 - 1)Pr_1^{-1}[k_G\bar{c}_1 + k_V S_1],$$

$$\left(E_1 = \gamma_1[\bar{c}_1^2 + G(\bar{l} - \bar{c}_1)^2], \quad E_2 = k_G\bar{c}_1 + k_V\gamma_1 G\bar{l}^2, \quad G = \left[\frac{\lambda_1\bar{c}_1 r}{\lambda_2\bar{c}_2}\right]^{1/2},\right.$$

$$\left. S_1 = \gamma_1\left[\bar{c}_1^2 + \frac{\lambda_1}{5\lambda_2}(\bar{l} - \bar{c}_1)^2\right], \quad S_2 = k_G\bar{c}_1 + \gamma_1\bar{l}^2 k_V\frac{\lambda_1}{5\lambda_2}\right). \tag{4.4}$$

Here $a_{i,j}$ are the averaged radii determined by the following:

$$a_{i,j} = \left[\frac{\langle a^i\rangle}{\langle a^j\rangle}\right]^{1/(i-j)} \quad \langle a^i\rangle = \int_{a_{\min}}^{a_{\max}} N_0(a)a^i\,\mathrm{d}a, \quad i \neq j, \quad a_{\min} < a_{i,j} < a_{\max}\,. \tag{4.5}$$

The first components in the coefficients $K^{(1)}_\infty$ and $K^{(2)}_\infty$ (i.e., 3/2 and 1) are related to the interface friction, the rest are related to the interface heat and mass exchange. In this case the second terms in E_i and S_i (i = 1, 2) are related to the non-homogeneous distribution of the temperature within the droplets. The analysis shows that for the typical cases ($\lambda_1 < \lambda_2$, $\bar{l}^2 \gg 1$, $r \ll 1$) these terms can make an essential contribution to the coefficients $K^{(1)}_\infty$ and $K^{(2)}_\infty$. But for suspensions with no phase transitions the effect of non-uniform temperature distribution is insignificant (Nigmatulin, 1990).

According to (4.3) and (4.4) the account of the effect of spectral composition on the propagation of high frequency disturbances ($\omega\tau^\circ{}_v \gg 1$) in the polydisperse vapor–gas-droplet mixtures with any form of the distribution function $N_0(a)$ comes through the integral characteristics $a_{3,2}$ and $a_{3,1}$. It means that this case is similar to gas-particle suspensions with no mass exchange. At frequencies $\sqrt{r\omega\tau^\circ{}_v} \gg 1$ the attenuation coefficient, K_{**}, is generally determined by the main asymptotic term with the coefficient $K^{(1)}_\infty$. Then, if $a^\circ = a_{3,2}$ is assumed, relationship (4.3) and (4.4) coincides with the corresponding asymptotic for the monodisperse suspension.

It should be noted that taking into account the actual non-equilibrium state of the interface mass exchange the main term of high frequency asymptotic ($\omega \to \infty$) for aerosols is to coincide with that of the corresponding asymptotic for suspensions with no mass exchange (Gumerov and Ivandaev, 1988), since phase transitions have simply no time to take place. Such a relationship can be obtained from the asymptotic (4.3) and (4.4) at $k_V = 0$.

Expression (4.4) for the coefficient $K_{\infty}^{(1)}$ may be modified (Gubaidullin and Ivandaev, 1992):

$$K_{\infty}^{(1)} = K_{\infty}^{(1G)} + (\gamma_1 - 1)Pr_1^{-1/2}\frac{\gamma_1 k_V \bar{c}_1[\bar{c}_1(1+G) - 2\bar{l}G]}{(1-k_V)\bar{c}_1 + k_V l^2 \gamma_1 G},$$

$$K_{\infty}^{(1G)} = \frac{3}{2} + (\gamma_1 - 1)Pr_1^{-1/2}$$

where $K_{\infty}^{(1G)}$ is the coefficient of the first (main) terms of the asymptotic in the absence of mass transfer (Gumerov and Ivandaev, 1988). For the mixture of air with vapor and water droplets we may accept that $\bar{l}G \ll 1$, then $K_{\infty}^{(1)} > K_{\infty}^{(1G)}$, and the difference $K_{\infty}^{(1)} - K_{\infty}^{(1G)}$ increases with growth of the vapor concentration k_V, that leads to overrating the attenuation coefficient, K_{**}. This is connected with the neglect of the real non-equilibrium state of evaporation and condensation. Note that the equilibrium phase transition approximation for the attenuation coefficient, K_{**}, may be used when (Nigmatulin, 1990; Gubaidullin and Ivandaev, 1987):

$$\left(\frac{\lambda_2}{\lambda_1}\right)\left(\frac{C_1^2}{l}\right)\left(\frac{L_m}{a_{3,2}}\right)[\omega\tau_{\lambda_2}(a_{3,2})]^{1/2} \ll 1, \quad L_m = \frac{\kappa_1}{C_1},$$

where L_m is the mean free path in gas ($L_m \sim 10^{-8}$ m). For high frequency perturbations, when the last condition is invalid, the phase transition equilibrium approximation leads to overrating the attenuation coefficient. In this case the error increases with the increase in vapor concentration or with the increase in the initial temperature, T, for the fixed pressure, p.

The low-frequency asymptotic for K_*, describing the dependence of the attenuation coefficient K_{**} for the mixture with a small mass droplet content ($m^3 \ll 1$, $\alpha_2 \ll 1$) and small frequencies ($\omega\tau^{\circ}_v \equiv \Omega \ll m$) has the form

$$\frac{\bar{K}_*}{\Omega} = (\bar{C}_e)^{-1} + \frac{i}{2}\left\{mF\bar{a}_{5,3}^2 + \frac{k_V}{m}\theta\bar{a}_{3,1}^2\right\} + O\left(m^3, \frac{\Omega}{m}\right),$$

$$F = 1 + \frac{3}{2}Pr_1(\gamma_1 - 1)\left[(1-k_V)\left(\frac{c_2}{c_1}\right)\right]^2, \quad \theta = \frac{\frac{3}{2}Pr_1(\gamma_1 - 1)(\bar{l} - \bar{c}_1)^2\gamma_1}{\gamma_1 \bar{l}^2 k_V + (1-k_V)\bar{c}_1}, \quad \bar{C}^{(e)} = \frac{C^{(e)}}{C_1}. \tag{4.6}$$

where $\bar{C}^{(e)}$ is the dimensionless equilibrium ($\Omega = 0$) sound speed in the gas–vapor-droplet mixture, which may be expressed for small mass droplet content ($m^2 \ll 1$) by the following:

$$\frac{1}{\bar{C}^{(e)}} = 1 + \frac{1}{2}\left\{ m\left[1 + (1 - k_V)(\gamma_1 - 1)\frac{c_2}{c_1}\right] + \frac{\gamma_1 k_V(\gamma_1 - 1)(\bar{l} - \bar{c}_1)^2}{\gamma_1 \bar{l}^2 k_V + (1 - k_V)\bar{c}_1} \right\}. \tag{4.7}$$

In Eq. (4.6) the first component of the coefficient F, which is equal to one, determines the effect of the interface friction, the second component is related to the temperature relaxation of the 'inert' gas taking no part in the interface mass exchange. The term with the coefficient θ determines the contribution of phase transition effects to wave dissipation.

It should be emphasized that in accordance with (4.6) the contribution of the interface friction and interface heat and mass exchange in dissipation of waves are additive in aerosols.

For the small mass droplet content ($m \ll 1$) when the vapor concentration is larger than the liquid content ($k_V > m$) the term θ dominates because of the multiplier $1/m$, and the propagation of low frequency disturbances in the polydispersed fogs is governed mainly by the interface mass exchange effects.

For the low-frequency disturbances ($\omega \tau^{\circ}{}_v \ll m$) in correspondence with asymptotics (4.6) the account for polydispersion composition comes only through two integral characteristics of spectral composition of the mixtures: $a_{3,1}$ and $a_{5,3}$. In this case the averaged radius, $a_{5,3}$, relates to the effects of the interface friction and the interface heat exchange (for $k_V \neq 1$) while the averaged radius, $a_{3,1}$, relates mainly to the effect of phase transitions. According to the Gender inequality we have $a_{3,1} < a_{5,3}$ (for monodisperse suspensions $a_{3,1} = a_{5,3} = a_0$, or $\bar{a}_{3,1} = \bar{a}_{5,3} = 1$).

Thus, the damping of low-frequency ($\Omega \ll m \ll 1$) disturbances in polydispersed vapor–gas-droplet mixtures may be described approximately within the limits of the model with two effective averaged radii of droplets. When the mass exchange is absent ($k_V = 0$), or in the case when the effects of phase transition dominate, the acoustic properties of polydispersed fogs are described with the 'monodisperse model' selecting as a characteristic radius either $a_{5,3}$ or $a_{3,1}$, respectively.

5. On the condition of linear analysis applicability

It should be kept in mind that the linear solution may appear inadequate at small frequencies of the disturbances with the decrease of droplets mass content m for the suspensions with phase transformations. To make it applicable it is necessary to satisfy the condition $\Delta m/m \ll 1$ or $\Delta a/a \ll 1$, that imposing a stricter limitation on the pressure amplitude than $\Delta p_1/p_0 \ll 1$. Let us obtain this relationship. For that purpose let us put down the equation of heat in the flow interface surface (2.2) in the form:

$$n_0 j = \frac{1}{l_0}\left[\rho_{10} c_{\mathrm{p}_1} \frac{\partial T'_1}{\partial t} - \alpha_{10}\frac{\partial p'_1}{\partial t} + \rho_{20} c_2 \frac{\partial T'_2}{\partial t}\right] = 4\pi a_0^2 \rho^{\circ}{}_{20} n_0 \frac{\partial a'}{\partial t}.$$

Substituting the solution of form (2.6) we shall get a dimensionless relation between the pressure disturbance and that of the droplet radius in the wave at the mass exchange:

$$\frac{a'}{a} = -\frac{1}{3m\gamma_1 \bar{l}}\left[1 - \frac{\bar{c}_1 + m\bar{c}_2}{\bar{l}}\right]\frac{p'_1}{p_{10}}. \tag{5.1}$$

It is simple to see that for implementing the condition $a'/a \ll 1$, or $\Delta m/m \ll 1$ the following limitation on the amplitude of the pressure disturbance is necessary

$$\frac{p_1'}{p_{10}} \ll \frac{3m\gamma_1\bar{l}}{1-(\bar{c}_1+m\bar{c}_2)/\bar{l}}. \tag{5.2}$$

Since the matter of interest is the disturbances with 'appreciable' amplitude $p_1'/p_{10} > 10^{-2}$, and having in mind that usually $\bar{l} > 10$, it follows that for suspensions with phase transitions the linear analysis is effective for aerosols with droplet mass content $m > 10^{-3}$.

6. The influence of polydispersed composition on the acoustic properties of vapor–gas-droplet systems

Figs. 1–4 illustrate the numerical results on sound propagation in polydispersed suspensions calculated in a broad frequency range Ω, not only when low and high frequency asymptotics are applicable. The calculations are made by the dispersion relations (3.6) and (3.7) for the mixture of air with vapor and water droplets under the gas phase pressure $p_{10}=0.1$ MPa, temperature $T_0=327$ K, $k_V=0.1$ and various droplet mass contents m.

The dispersion curves are shown in the form of the dependence of the attenuation coefficients σ, K_{**} and that of the phase velocity $C^{(p)}$ on the dimensionless frequency $\Omega_{3,1}$ ($\Omega_{3,1} \equiv \omega\tau^{\circ}{}_v$, for $a^{\circ}=a_{3,1}$). Here the radius, $a_{3,1}$, is used as a scale parameter. In accordance with the asymptotics (4.6) for K_* the radius, $a_{3,1}$, is a characteristic averaged radius of

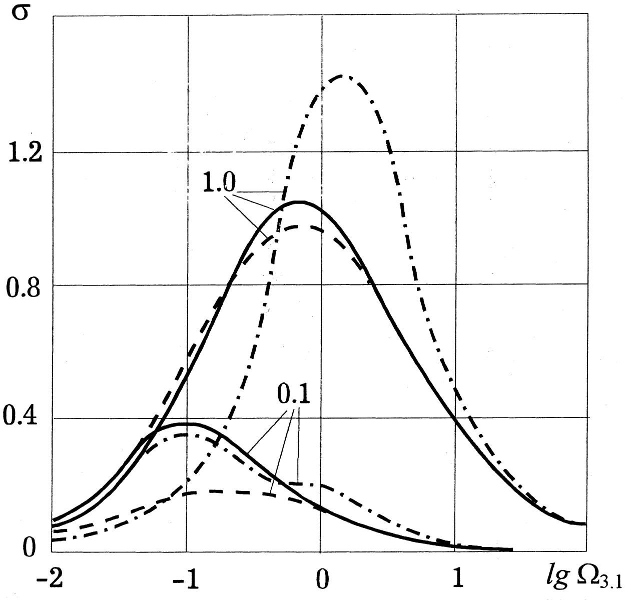

Fig. 1. Dimensionless attenuation coefficient per wavelength dependence upon oscillation dimensionless frequency at different droplets mass contents.

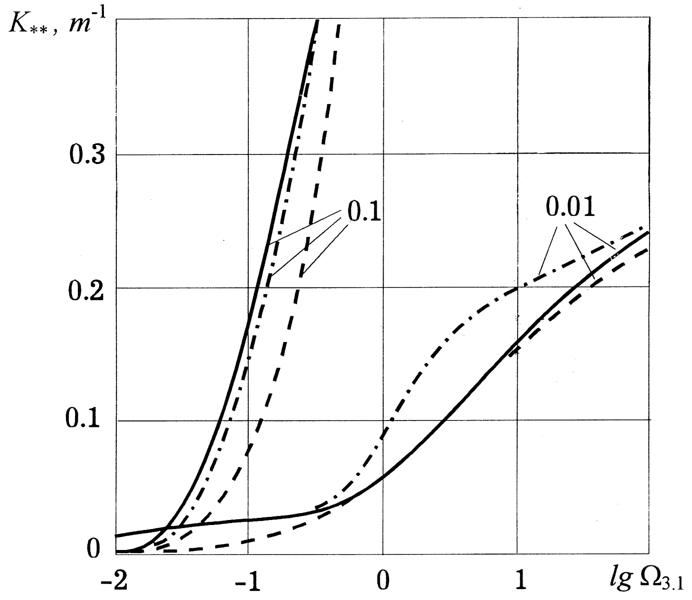

Fig. 2. Linear attenuation coefficient dependence upon oscillation dimensionless frequency at different droplets mass contents.

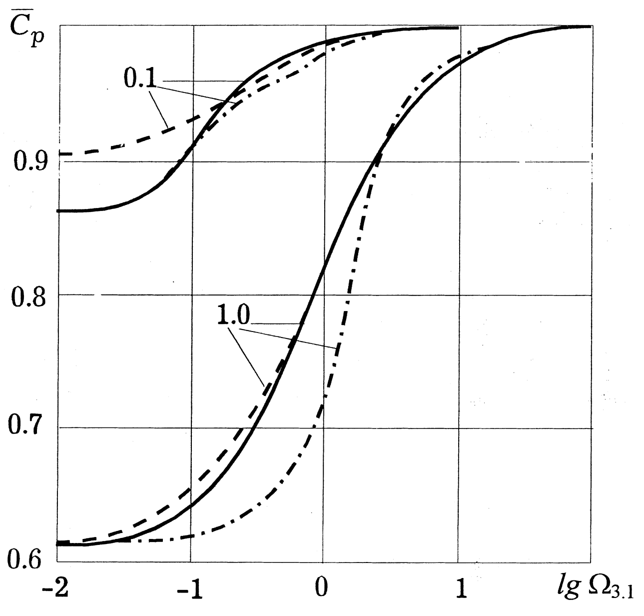

Fig. 3. Dimensionless phase velocity dependence upon oscillation dimensionless frequency at different droplets mass contents.

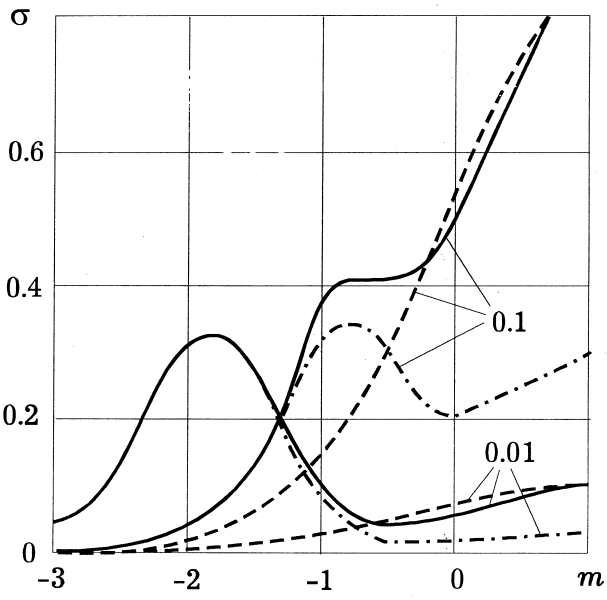

Fig. 4. Dimensionless attenuation coefficient per wavelength dependence upon droplets mass contents.

polydispersed vapor–gas-droplet mixture under low frequency acoustic effects. This radius is characteristic for high-frequency waves with no mass exchange too. In this case according to the Gelder inequality the value of $a_{3,1}$ is minimum among the values of the other considered averaged radii of droplets in the polydispersed suspension: $a_{3,1} < a_{3,2} < a_{5,3}$. That is why the application of another radius, e.g., $a_{5,3}$ as a scale parameter ($a^\circ = a_{5,3}$), leads to a shift of the dispersion curves along the x-coordinate towards the dimensionless frequency growing, $\eta \equiv \log \Omega$, since

$$\log\ \Omega_{5,3} = \log\ \Omega_{3,1} + 2\ \log(a_{5,3}/a_{3,1}), \quad (a_{5,3}/a_{3,1} \geq 1). \tag{6.1}$$

Further, to illustrate sound propagation in the polydispersed air fog with continuous droplet distribution an even distribution of droplets by masses is taken:

$$N(a) = A^\circ a^{-3},$$

$$(a_{\min} = 10^{-6}\ \text{m},\ a_{\max} = 10^{-5}\ \text{m}, a_{3,1} \approx 3 \times 10^{-6}\ \text{m}, a_{5,3} \approx 6 \times 10^{-6}\ \text{m}). \tag{6.2}$$

The value of the constant parameter A° may be determined by the value of the volume content of the droplets, α_{20}, or by the mass concentration m. However, according to (3.7) and (4.5) the value of A° does not influence the form of dispersion curves and the magnitude of the averaged radii $a_{i,j}$. In this case for the characteristic average radii $a_{3,1}$ and $a_{5,3}$ [see (4.5)] the following relationships take place:

$$a_{3,1} = a_{\min}\sqrt{\chi}, \quad a_{5,3} = a_{\min}\sqrt{\tfrac{1}{3}(\chi^2 + \chi + 1)}, \quad \left(\chi = \frac{a_{\max}}{a_{\min}}\right). \tag{6.3}$$

Figs. 1–3 show the dependencies of the dimensionless attenuation coefficient per wavelength, σ, linear attenuation coefficient, K_{**}, and phase velocity, $C^{(\mathrm{P})}$, on the dimensionless frequency, $\Omega_{3,1}$, at various droplet mass contents, m. The full and dash-lined curves (here and below) correspond to the cases of non-equilibrium ($\beta = 0.04$) and frozen ($\beta = 0$) mass exchange among drops and gas in the polydispersed fog. The dash–dot lines are used to illustrate the dispersion curves which correspond to the monodisperse vapor–gas-droplet mixture with the droplet radius $a_0 = a_{3,1} = 3 \times 10^{-6}$ m at the non-equilibrium phase transformation. The numbers near the curves show the mass content values of the suspended phase, m.

The analysis shows that for extremely small mass content of the droplets, $m \leq 0.01$, the form of the dependence $\sigma(\Omega_{3,1})$ for the polydispersed fog is governed mainly by mass exchange and practically coincides with the corresponding dependence for the monodisperse mixture (Gubaidullin and Ivandaev, 1987). As the mass droplet content grows the contribution of the interface friction to the disturbances dissipation increases, and at $m \sim 0.1$ the full and dash–dot curves differ. This difference is the most essential at the average frequencies $\Omega_{3,1} \sim 1$, when the influence of the polydispersed mixture composition upon sound propagation is maximum and cannot be described in terms of monodisperse models applying the effective radii (Figs. 1–3). That is why for this frequency band neither of the average radii (4.5) can be used as a characteristic dimension. It should be noted that under sufficiently high mass droplet contents, $m \sim 1$, the damping factor value σ for a monodisperse aerosol may be both considerably exceeding and considerably lower than the value σ for a polydispersed suspension under different frequencies $\Omega_{3,1}$. The influence of the interface mass exchange effects upon the wave propagation in the polydispersed fogs, as well as for monodisperse suspensions, is expressed mostly in the aerosols with small m (full and dashed curves, Figs. 1–3). With m increasing the role of multi-velocity (slip velocity) effects grows and the difference between the curves shown diminishes.

7. The effect of non-monotonous dependence of sound dissipation on drops concentration in aerosol acoustics

The essential influence of phase transformations on low frequency disturbance propagation in the polydispersed aerosols with small droplet mass content, m, leads to an anomalous effect of non-monotonous dependence of sound attenuation on the droplet contents, m. This effect is quite unexpected and opposite to the generally accepted point of view holding that the disturbance damping intensity in such systems is proportional to mass content of droplets, which are the source and main cause for dissipation.

First, let us consider acoustics of monodisperse gas–vapor-droplet suspension with the radius of the droplets, a_0. Used are characteristic times of velocity, τ_{v_0}, and temperature, τ_{T_0}, relaxation of a single dispersed particle;

$$\tau_{v_0} = \frac{2}{9}\frac{\rho^{\circ}{}_{20} a_0^2}{\mu_1}, \quad \tau_{T_0} = \frac{\rho^{\circ}{}_{20} c_2 a_0^2}{3\lambda_1}. \tag{7.1}$$

Note that the times of relaxation (leveling in phase velocities and temperatures) in the two-phase mixture depend on mass concentration of droplets m. To evaluate the influence of m on the process of relaxation let us consider a certain volume of vapor–gas-droplet mixture at $t < 0$ in the initial thermodynamically equilibrium state ($v_1 = v_2 = 0$, $T_1 = T_2 = T_\Sigma = T_e$, $p_1 = p_e$).

Suppose that at $t = 0$ in response to the external action the parameters of the gas phase in every point of the volume changed suddenly by one and the same value, whereas the parameters of droplets remain as before, corresponding to the initial equilibrium state. In the resulting non-equilibrium vapor–gas-droplet mixture there will take place the relaxation processes of interphase volume mass, pulse and energy, owing to which the parameters of the mixture at $t \to \infty$ will tend (relaxate) to their new equilibrium values ($v_1 = v_2 = v_\infty$, $T_1 = T_2 = T_\Sigma = T_\infty$, $p_1 = p_\infty$).

It is known that any relaxation process is described by the equation like:

$$\frac{\mathrm{d}\psi(t)}{\mathrm{d}t} = -\frac{1}{\tau}[\psi(t) - \psi(\infty)], \tag{7.2}$$

with the solution having the form:

$$\psi(t) - \psi(\infty) = \psi'(t) = (\psi(0) - \psi(\infty)) \exp(-t/\tau), \tag{7.3}$$

where $\psi(t)$ is the variable dependent on the time and taking the equilibrium values $\psi(\infty)$ at $t \to \infty$; ψ' is its disturbance relative to the equilibrium state $\psi(\infty)$, τ is the relaxation time characterizing the process of exponential approximation of the parameter ψ to its equilibrium value $\psi(\infty)$. To determine the relaxation time τ it is possible to examine the solution of the system of the linearized equations (2.7) having the form (7.3) for the disturbance of the parameters:

$$\{\psi'\} = \{\rho_1', \rho_V', \rho_2', v_1', v_2', p_1', T_1', T_2', T_\Sigma'\}.$$

Thus, a simplified formulation of the problem is considered on the typical exponential relaxation process in the vapor–gas-droplet mixture. A relevant solution may also be obtained from the dispersion relation for $K_*(\omega)$ by means of substitution $i\omega = 1/\tau$, $K_* = 0$. The desired relaxation times are the roots of a certain algebraic equation derived in such a manner—the square root for the suspension with no phase transformations and the cube root for the general case of vapor–gas-droplet mixture.

There are two such time points for gas suspensions with no phase transformations, and they are related with times (7.1) by relationships (Gubaidullin and Ivandaev, 1991):

$$\tau_1 = \frac{t_{v0}}{1 + m}, \quad \tau_2 = \frac{\tau_{T0}}{1 + mc_2/c_{V_1}}. \tag{7.4}$$

The vapor–gas-droplet systems with phase transformations have three characteristic relaxation times independent from each other, in this case, τ_1, determines the interface momentum exchange, τ_2 and τ_3 characterize the interface heat and mass exchange. They depend on vapor concentration in the gas phase, vaporization specific heat and many other thermophysical phase parameters.

Because of that it is difficult to get any evident expressions for these times in the general case.

In the particular case with a small droplet mass content, m, and extreme vapor concentration, k_{V_0} (small and high values correspondingly) the problem is simplified. The estimations show that at small k_{V_0} in the general case of vapor–gas-droplet mixture we have:

$$\tau_2 \sim \tau_T, \quad \tau_3 \sim \tau_{k_1} = \frac{\alpha_{10}}{3\alpha_{20}} \frac{a_0^2}{D_1}, \tag{7.5}$$

where τ_{k_1} and D_1 are the time and coefficient of vapor diffusion in the gas phase. In the case of high concentration of the vapor, k_{V_0}:

$$\tau_2 \sim \tau_{T_2}, \quad \tau_3 \sim \tau_{T_1} = \frac{\alpha_{10} a_0^2}{3\alpha_{20}\kappa_1}, \quad \kappa_j = \frac{\lambda_j}{\rho^{\circ}_{j0} c_j}. \tag{7.6}$$

Here τ_{T_2} is the relaxation time of the temperature in a droplet, τ_{T_1} is the relaxation time of the temperature in the gas phase, κ_j is the coefficient of the j-phase thermal diffusivity. As a rule:

$$\tau_{T_2} \ll \tau_v \sim \tau_T \quad \text{and} \quad \tau_{k_1} \sim \tau_{T_1} \sim \tau_v/m. \tag{7.7}$$

Note that the strong connection between the time, τ_3, and the volume droplet content, α_{20} ($\tau_3 \sim \tau_d/3\alpha_{20}$) has a certain physical sense and is related to the processes of the parameters equalization (temperature and vapor concentration) in the area (cell) around a droplet. The more numerical drops concentration, n_0, the less is the characteristic cell dimension $R \sim n^{-1/3}$, and, accordingly, the less is the temperature equalization and vapor concentration time, τ_3, in the area around a droplet. In the aerosols with small m time $\tau_3 \gg \tau_v$, τ_2. In this connection the great effect on disturbance damping may be expected with the frequencies, $\omega\tau_3 \sim 1$ ($\omega\tau_v \ll 1$).

In accordance with the dispersion relationship at the small mass droplets content $m \ll 1$ the following relationship may be obtained for the attenuation coefficient (Gubaidullin and Ivandaev, 1990):

$$\sigma \approx \pi \left(\frac{C^{(\mathrm{P})}}{C_1} \right)^2 \sum_{i=1}^{3} G_i \frac{\omega\tau_i}{1 + (\omega\tau_i)^2}. \tag{7.8}$$

Here $\tau_i(a)$ are the characteristic phase interaction time in the suspension, G_i, are the coefficients depending on phase thermophysical properties. Thus, for the vapor droplet aerosol with regard to $C^{(\mathrm{P})} \sim C_1$, $\tau_1 \sim \tau_v$, $\tau_2 \sim \tau_{T_2}$, $\tau_3 \sim \tau_{T_1} \sim \tau_v/m$, relationship (7.8) has the form (Nigmatulin et al., 1991):

$$\sigma = \pi \frac{m\omega\tau_v}{1 + (\omega\tau_v)^2} + (\gamma_1 - 1)\left(1 - \frac{\bar{c}_1}{\bar{l}} \right)^2 \frac{m\omega\tau_v}{m^2 + (\omega\tau_v)^2}. \tag{7.9}$$

According to expression (7.9) the dependence of the attenuation coefficient σ on the dimensionless frequency $\Omega = \omega\tau_v$ has two maxima for the monodisperse mixture. One of them is related to the dissipation effects because of phase transformations and occurs at the frequency $\Omega^{(1)} \sim m$ ($\omega\tau_{T_1} \sim 1$), the other is observed at $\Omega^{(2)} \sim 1$, and is conditioned by dissipation because of the interface friction. In this case, as $m \ll 1$ is small, the inequality $\Omega^{(1)} \ll \Omega^{(2)}$ takes place for the characteristic frequencies $\Omega^{(i)}$ (i = 1, 2).

In accordance with the asymptotics (4.6) at low frequency disturbances the formulas of monodisperse suspension acoustics may be applied to describe approximately the acoustic properties of polydispersed aerosols. In this case, the droplets velocity relaxation is determined by the average radius $a_{5,3}$, while the interface mass exchange effects are characterized by smaller average radius $a_{3,1}$, so the characteristic phase interaction time τ_i are represented in the form of the corresponding functions: $\tau_v = \tau_v(a_{5,3})$, $\tau_{T_1} = \tau_{T_1}(a_{3,1})$. Then, according to (7.9), the first maximum of the coefficient σ, due to the processes of phase transformations in the polydispersed aerosols, will be observed at the frequency $\Omega^{(1)}_{3,1} \equiv \Omega^{(1)}(a_{3,1}) \sim m \ll 1$. According to (7.9) the interface friction contribution is at its maximum at the frequency $\Omega^{(2)}_{3,1} \sim (a_{3,1}/a_{5,3})^2 \ll 1$, $(\Omega^{(2)}(a_{5,3}) \sim 1)$. Because of the frequency $\Omega^{(1)}$ and $\Omega^{(2)}$ values being proximate, the individual maxima related to the interface friction and interface mass exchange are not separated, and the dependence $\sigma(\Omega)$ has the only maximum (Fig. 1) in the polydispersed case in question.

In accordance with (7.9) the attenuation coefficient dependence σ on the droplet mass contents m both monodisperse and polydisperse aerosols at the frequency $\omega\tau_v \ll 1$ $(\Omega_{3,1} \ll 1)$ is not monotone and has its maximum at $m \sim \omega\tau_v$ $(m \sim \Omega_{3,1})$, related to the dominating influence of the heat mass exchange effects.

Fig. 4 shows the characteristic form of the attenuation coefficient dependence σ on the droplet mass content at fixed dimensionless frequencies: 1. $\Omega_{3,1} = 0.01$ $(\omega \sim 100\ \mathrm{s}^{-1})$, 2. $\Omega_{3,1} = 0.1$ $(\omega \sim 1000\ \mathrm{s}^{-1})$. It can be seen that the dependence $\sigma(m)$ is non-monotonous and has its local maximum at the value $m \sim \Omega_{3,1}$. However, for the monodisperse fog the maximum σ at frequency $\Omega_{3,1} = 0.1$ is expressed more obviously (dash–dot curve). Let us note that at $m \ll 1$ the damping intensity in the suspension with phase transformations essentially exceeds that in the gas with particles where these transformations are absent (dashed curves). With m increasing the influence of the interface mass exchange effects upon damping of the frequency disturbances in question diminishes (full and dashed curves coincide in practice at $m \gg 1$). In this case the disturbances damping in the polydispersed gas suspensions with high droplet mass content $m \sim 10$ may considerably exceed (three times as much) the wave dissipation in the monodisperse systems (full and dash–dot curve 2).

Thus, for low frequency disturbances $(\omega\tau_v < 0.1)$ the damping intensity in the aerosols with the mass exchange depends non-monotonously on the dispersed phase mass content, m. At m being small enough (usually when $m < 1$), the exact values of which depend on frequency and phase thermophysical parameters there is the local damping maximum observed due to various contributions of the interface heat mass exchange and phase friction to wave damping at different frequencies.

8. Conclusion

Phase transitions in polydispersed gas–vapor-droplet suspensions (fogs) for relatively a small frequency wave may lead to a non-monotonous dependence of the wave attenuation on the mass concentration of droplets, which are the main cause for wave absorption.

For polydispersed fogs with a small mass fraction of droplets the contributions of the interface friction, heat and mass exchange in the wave absorption are additive. However, unlike the suspensions with no phase transitions the interface heat and mass exchange contribution is not proportional to mass concentrations of the droplet fraction with the different radius. It means that unlike the suspensions with no phase transitions the corresponding dispersion function cannot be obtained by the integration of the dispersion function for the monodisperse suspension by mass of the droplet fraction.

Among the dissipation effects for low frequency waves in the polydispersed suspensions with small mass concentration of droplets the effects of the interface heat and mass exchange dominate. The attenuation coefficient for relatively low frequencies may be estimated by the model for a suspension with two radii of the droplets. One of them is connected with the interface friction, and the other with the phase transitions.

Propagation and attenuation of the relatively high frequency waves in the polydispersed gas–vapor-droplet suspensions may be described in the frame of a model of the monodisperse suspension with some averaged droplet radius, which is an integral characteristic of the droplet distribution function by size.

For the high frequency waves the approximation of the equilibrium phase transitions leads to overestimation of the attenuation coefficient. The error because of ignoring the non-equilibrium phase transitions increases with the growth in vapor concentration, or, what is the same, with the growth in the temperature of the suspension for the fixed pressure.

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